

Tweedledee's Logic: Squaring *Reductio ad Absurdum*.

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Published in *The Carrollian: The Lewis Carroll Journal*, May 2019, Issue 32, pp. 3-26.
www.thecarrollian.org.uk

1. Introduction

In 2017 I wrote an article on Lewis Carroll's use of *reductio ad absurdum* in his *Alice* books [Savenije 2017]. In the article I cited the famous argument of Tweedledee about logic:

“Contrariwise,” continued Tweedledee, “if it was so, it might be, and if it were so, it would be but as it isn't, it ain't. That's logic.”¹

It is a citation from *Through the Looking-Glass and What Alice Found There*. Tweedledee is addressing Alice who has entered the world behind a mirror and now meets the Tweedle twins, Tweedledum and Tweedledee.

Although this quote has at least a resemblance with *reductio ad absurdum*, I decided that it was outside the scope of my article. Its analysis would have created an unbalance in the article, since I would have to deal with the different interpretations that can be found in the literature.

In the present article I intend to make up for this. I will present an analysis of Tweedledee's argument from a logical point of view, taking into account the different interpretations I found in literature as well as in translations into Dutch, German and French.

The structure of my analysis is determined by the fact that these different interpretations can be traced back to differences in assessing several elements of Tweedledee's argument:

- The meaning of the word 'contrariwise'. It is used five times by Tweedledee in the chapter of *Through the Looking-Glass* in which the quote occurs. For a proper understanding it appears to be helpful to take a closer look at the relation between the Tweedle twins, since Tweedledee is using 'contrariwise' in interaction with his brother.
- The status of Tweedledee's argument in the light of his own qualification at the end of the citation: 'That's logic'.
- The meaning of Tweedledee's use of the different modalities of the verb *to be* ('was', 'might be', 'were so', 'is') in the argument.
- The reference of the six occurrences of 'it' in Tweedledee's argument.

On the basis of an analysis of these elements I will evaluate the argument itself.

2. The Tweedle Twins²

In *Through the Looking-Glass and what Alice Found There* Alice climbs up onto the fireplace mantel and steps through the mirror on the wall into an alternative world in which everything is inverted. Here she meets Tweedledum and Tweedledee and “the

words of the old song [keep] ringing through her head”³. Here Lewis Carroll uses the Tweedledum & Tweedledee nursery rhyme in the following form:

Tweedledum and Tweedledee
Agreed to have a battle;
For Tweedledum said Tweedledee
Had spoiled his nice new rattle.

Just then flew down a monstrous crow,
As black as a tar-barrel;
Which frightened both the heroes so,
They quite forgot their quarrel.

Carroll did not invent this rhyme himself. The rhyme was included in *Original Ditties for The Nursery; so Wonderfully Conceived that They May Be Either Sung Or Said, by Nurse Or Baby* from 1807⁴. There are, however, some differences with Carroll’s version. Only three copies of this work are known. During the time that Lewis Carroll wrote *Through the Looking-Glass* there was a copy in the Bodleian which was a seven-minute walk from Christ Church where Carroll lived.

The rhyme also occurred in a book that Lewis Carroll had in two versions in his own library: James Orchard Halliwell’s *The Nursery Rhymes of England, Collected principally from Oral Tradition*. This version also shows some differences with the other versions. When examining Carroll’s version of the Tweedle rhyme, the rational conclusion is that he must have consulted either of the versions mentioned, or quite likely both.

These rhymes are not the first appearances of the words ‘Tweedledum’ and ‘Tweedledee’. Usually an epigram written by John Byrom (1692-1763) is referred to as the oldest appearance in print. In this epigram he is satirising the disagreements between the composers George Frederick Handel and Giovanni Battista Bononcini (Byrom 1773, pp.343-344).

However, in The London Journal of June 5, 1725 we find an earlier occurrence of ‘Tweedledum’ and ‘Tweedledee’:

The CONTEST
By the Author of the celebrated Pastoral,
My Time, O ye Muses, was happily spent.

Some say that Seignior *Bononchini*,
Compar’d to *Handel’s* a meer Ninny;
Others aver, to him, that *Handel*
Is scarcely fit to hold a Candle,
Strange ! that such high Disputes shou’d be
“Twixt *Tweedledum* and *Tweedledee*.

This is also by Byrom who was the author of a pastoral of which the first line is “My Time O ye Muses, was happily spent.” Here we have the first known use in print of ‘Tweedledum’ and ‘Tweedledee’ and John Byrom is often said to have coined these names. However, we cannot be certain whether the nursery rhyme about the Tweedle

brothers referred to this famous musical battle, or whether it was an older rhyme from which Byrom borrowed in the last line of his doggerel [Gardner 2015, p.213].

The names 'Tweedledum' and 'Tweedledee' obviously revolve around a common onomatopoeia (sound imitation): to tweedle is to scratch an instrument, to produce a series of sharp sounds [Cixious & Maclean 1982, p.233]. John Byrom's poem is making fun of two quarrelling composers (Handel and Bononcini) in a poem that satirised and mocked two rivalling schools of music at the time. The poem was devised to highlight the petty disagreements between two musicians and their followers, with the names designed to suggest that very little actually separated the two factions in practice.

Consequently, the names 'Tweedledum' and 'Tweedledee' have become synonymous for any two people who differ only slightly and certainly not as much as they would have us believe. And this is how we recognise them in Lewis Carroll's story.

In chapter 4 of *Through the Looking-Glass* Alice meets the twins Tweedledum and Tweedledee who stand side by side with their arms around each other's shoulders. Upon seeing them, Alice begins to recite the poem she knows about Tweedledum and Tweedledee, which describes the fight about a broken rattle. The twins deny that this has ever happened. At the end of the chapter, however, they agree to have a fight about the ownership of a rattle, but the battle never takes place: before it begins, a great crow comes and scares them off, and Alice slips into the wood alone.

Alice calls the twins a "couple of great schoolboys". They act as complementary rivals, they work by doubling and overbidding [Cixious & Maclean 1982, p.233]. Florence Becker Lennon recognises them as "a special university type – querulous, meticulous, infantile, quarrelsome" [Becker Lennon 1945, p. 183].

John Tenniel pictured Tweedledum and Tweedledee as twins. According to Gardner [2015, p.215] they are "enantiomorphs", mirror-image forms of each other. He concludes that Carroll intended this from the fact that they extend right and left for a handshake, but also from Tweedledee's favourite word, 'contrariwise'.

3. Contrariwise

Inversion themes occur throughout all Carroll's literary writing. In real life Carroll also milked the notion of inversion as much as he could to amuse his child-friends. He liked to write in reverse and send letters which could only be read when they were held up to a mirror [Skinner 1949, p.297].

When Alice crosses the mirror from side to side, she comes into an alternative world, which reverses accepted patterns. The story of *Through the Looking-Glass* is a story of complete reversal of the real world, "literally considering things from the very opposite of the conventional point of view" [Polhemus 1980, p.291]. This includes the inversion of time and of causality, with effect preceding cause [Cixious & Maclean 1982, p.238].

In a mirror all asymmetrical objects (objects not superposable on their mirror images) "go the other way" [Gardner 2015, p.166]. If we consider the twins as mirror images,

this, indeed, throws light on their conversation: they are looking at things from a contrasting point of view.

Let us now look at the texts in which the word is actually used in *Through the Looking-Glass*.

First passage in which 'contrariwise' occurs:

"If you think we're wax-works," he [Tweedledum] said, "you ought to pay, you know. Wax-works weren't made to be looked at for nothing. Nohow!"

"Contrariwise," added the one marked 'DEE', "if you think we're alive, you ought to speak."

The question here is whether the twins are waxworks or alive. Tweedledum says that Alice has to pay to look at them, if she thinks they are waxworks. Tweedledee's remark is an addition to Tweedledum's "Nohow!", not a negation of it. He considers the situation from the point where Alice supposes that they are alive, and 'contrariwise' is used in the meaning of 'looking at it from the other side', 'suppose the opposite were true'.

The second passage:

"I know what you're thinking about," said Tweedledum; "but it isn't so, nohow."

"Contrariwise," continued Tweedledee, "if it was so, it might be; and if it were so, it would be but as it isn't, it ain't. That's logic."

As in the first example, Tweedledee gives an addition to Tweedledum's "nohow". There is no negation or contrast with Tweedledum. Where Tweedledum says "it isn't so", Tweedledee looks at the situation 'if it was so'. If there is a contradiction, it is with Alice's supposed point of view, that the twins are two figures from a nursery rhyme, which has already been denied by Tweedledum.

Tweedledee's 'contrariwise' then means 'looking at it from the other side'.

The third passage:

They looked so exactly like a couple of great schoolboys, that Alice couldn't help pointing her finger at Tweedledum, and saying "First Boy!"

"Nohow!" Tweedledum cried out briskly, and shut his mouth up again with a snap.

"Next Boy!" said Alice passing on to Tweedledee, though she felt quite certain he would only shout "Contrariwise!" and so he did.

"You've begun wrong!" cried Tweedledum. "The first thing in a visit is to say 'How do you do?' and shake hands." And there the two brothers gave each other a hug, and then they held out the two hands that were free, to shake hands with her.

Alice did not like shaking hands with either of them first, for fear of hurting the other one's feelings

'First Boy' was a term used in British schools for the brightest boy in a class, or an older boy who served a sort of class monitor.⁵ Alice addresses Tweedledum as "first boy" (to which he reacts with "Nohow!") and Tweedledee as "next boy", who then shouts "contrariwise". "You've begun wrong", is Tweedledum's explanation and the boys hug each other, indicating that they oppose Alice's distinction between first and next boy.

Tweedledee again uses his brother's expression 'Nohow' and opposes Alice's way of addressing them. 'It is the opposite way', might be the interpretation of this case, in the sense of denying that there is any ranking.

The fourth passage:

"I like the Walrus best," said Alice, "because he was a *little* sorry for the poor oysters."

"He ate more than the Carpenter, though," said Tweedledee. "You see he held his handkerchief in front, so that the Carpenter couldn't count how many he took: contrariwise."

"That was mean!" Alice said indignantly. "Then I like the Carpenter best – if he didn't eat so many as the Walrus."

Here Tweedledee confirms his own argument, opposing Alice's remark that the Walrus is to be preferred to the Carpenter because he would have eaten less oysters than the Carpenter. He indicates that 'the opposite is true', referring to Alice's remark. There is no direct interaction with his brother in this case.

The fifth passage:

"Do you think it's going to rain?"

Tweedledum spread a large umbrella over himself and his brother, and looked up into it. "No, I don't think it is," he said: "at least – not under *here*. Nohow."

"But it may rain *outside*?"

"It may – if it chooses," said Tweedledee, "we've no objection. Contrariwise."

As in the fourth passage, Tweedledee confirms his own argument. He opposes Alice's possible suggestion (formulated by himself) that he might have any objection to the possibility that it might be raining outside. He more or less reacts to his own thoughts without interaction with Tweedledum.

'On the contrary', or 'the opposite is true' is the meaning of 'contrariwise' in this case.

We can conclude that, in line with the traditional use of the names 'Tweedledum' and 'Tweedledee', there is no real contrast between the twin brothers. They never contradict each other, they even, according to the rhyme, "agree to have a battle". Rather, they complement each other's words.

Of course, superficially, Tweedledee's use of 'contrariwise' may give the impression that he is opposing his brother and probably that is why several commentators suggest that there is such a contrast.

For instance: "Tweedledee and Tweedledum look exactly alike, but no matter what the first says the other shouts: 'Contrariwise!' [...]. To every positive statement his brother makes, Tweedledum bellows, 'Contrariwise!' [...] Tweedledum and Tweedledee [...] are total denials of each other". "They are intimately connected by their disagreements. They always agree to disagree." [Odessey 2013]

Also, some Dutch translation show a contrast between the brothers, where "contrariwise" is translated as "Helemaal niet" (English equivalent: "Not at all") [Carroll 2016a, p.132] and "Ik ben het er niet mee eens" (English equivalent: "I do not agree") [Carroll 1964, p.148].

Jourdain [1918, p.11] ascribes “an opposition to authority” to Tweedledee, because of his frequent use of the word “contrariwise”. However, I can find no evidence for this. Rather, the conversation between the twins displays a symmetrical position. ‘Contrariwise’ functions as a transitional word that flips the premise of the conversation (SparkNotes Editors 2005). In three cases Tweedledee complements his brother and addresses the other side of whatever Tweedledum just said. In two cases Tweedledee confirms his own argument referring to a (supposed) suggestion of Alice. The meaning of ‘Contrariwise’ can then be characterised as: ‘Looking at it from the opposite side’ or ‘It is the opposite way’, but never as a negation of his twin brother.

4. Defining logic

Tweedledee concludes his argument with the words “That’s logic”. These words give rise to multiple interpretations, as becomes clear from different translations of *Through the Looking-Glass*. Comparing translations in Dutch, German and French, we see that several translators translate “That’s logic” as if Tweedledee had said: “That’s logical”.

Here are some examples.

Among the Dutch translations we find “Logisch toch?” (in English: “Isn’t that logical?”) (Carroll 2014, p.69), but also “Dat is nu logica” (Carroll 1956, p.136) and “Dat is logica” (Carroll 2016b, p.205).

This is comparable with some German variants: “Das ist Logisch” (Carroll 1948, p.60; 2015, p.40) and “Das ist Logik” (Flemming 2013, p.229).

In French we find “C’est logique” (Carroll 1950, p.121; 1954, p.156), “N’est-ce pas logique?” (Carroll 2005, p.126) but also “... en bonne logique” (Carroll 1989, p.160).

There is, however, no good reason to interpret ‘That’s logic’ as ‘That’s logical’. This is supported by Milner’s edition of *Through the Looking-Glass* [1936, p.197] where she explains in a footnote the meaning of the word *Logic* as “The science of reasoning”. Moreover, in many books about philosophy or logic citations can be found of Tweedledee’s statement as a statement *about logic*. Often it is a citation in a chapter about logic without an explicit relation with the text of the chapter⁶.

Sometimes we do find a short explanation or context.

“What Tweedledee is doing – at least in Carroll’s parody – is reasoning. And that, as he says, is what logic is about” [Priest 2000, p.1].

Or “We shall be occupied in this book with the philosophy of logic in substantially Tweedledee’s sense of the word ‘logic’” [Quine 1970, p.xi].

There is a real difference between ‘That’s logic’ and ‘That’s logical’. ‘That’s logical’ is a qualification of Tweedledee’s statement as being sound reasoning. ‘That’s logic’, however, it is not so much a characterisation of Tweedledee’s statement, but it rather states that this is what logic is about: Tweedledee is defining logic.

According to Quine [1970, p.11], Tweedledee describes logic by way of an example of reasoning. Tweedledee is giving a so-called ostensive definition. He specifies the meaning of ‘logic’ by pointing to an example to which logic applies, “an example of (alleged) logical reasoning” (Finocchiaro 2012, p.339).

This is a definition of logic in the sense of logical practice or argumentative reality which is the subject matter of logical theory, the systematic study of logical truths. This is

consistent with the definition of logic given by Carroll, probably in 1894: “The science of reasoning rightly” [Abeles 2010, p.75]⁷.

5. Analysing the structure of Tweedledee’s argument

Let us examine how the words of Tweedledee “If it was so, it might be, and if it were so, it would be but as it isn’t, it ain’t” qualify as an argument.

An argument is a set of statements formulated in such a way that one or more of them (‘premise’ or ‘premises’) are claimed to be proof of, or evidence for, another of the statements (‘conclusion’). In trivial arguments, the conclusion simply repeats one of the premises.

In logic we are interested in whether the conclusion follows from the premises, rather than whether the conclusion is true.

When we break down the citation in the following way, it becomes clear that Tweedledee is presenting an argument:

Premises: 1. If it was so, it might be.
 2. If it were so, it would be.
 3. It isn’t.
Conclusion: 4. It ain’t.

In order to analyse the structure of the argument in greater detail, it is useful to translate the sentences 1-4 into a logical form. This requires special attention to the following questions:

- what is meant by ‘is’ and the other variants of the verb ‘to be’?
- what does ‘it’ stand for?

On the basis of the grammatical form of the sentences 1-4 mentioned above, we can easily rule out the following possibilities for the meaning of ‘is’:

- a. ‘is’ in the sense of an identity, as in the statement ‘subject S_1 is identical to subject S_2 ’⁸;
- b. ‘is’ in the sense of a copula, attributing a predicate to a subject, as in the statement: ‘ S has the property P ’ or ‘ S belongs to the class of P ’ (a predicative statement);
- c. ‘is’ in the sense of existence where S is a subject: ‘ S exists’.

Consequently, we can also rule out the possibility that ‘it’ refers to a subject.

We may conclude that Tweedledee uses ‘is’ in the sense of ‘being the case’ or ‘being true’, while ‘it’ stands for any statement, being a declarative sentence that is either true or false.

This means that we are talking about propositional logic, the logic in which statements are the basic units.

Premises 1 and 2 have the form of hypotheticals (“if – then”) and here Tweedledee uses different moods of the verb ‘to be’.

Both the ‘if’-clauses use a subjunctive mood of the verb ‘to be’ (‘was so’, ‘were so’); both the ‘then’-clauses use an auxiliary verb (‘might’, ‘would’).

There is a difference between ‘was’ in sentence 1 and ‘were’ in sentence 2. In conditional sentences like ‘if it *was* so’, the speaker refers to a state of affairs (past or present) whose reality is accepted for the time being. However, ‘if it *were* so’ refers to a condition which is contrary to fact, but only in this case is supposed to be true [Curme 1953].

Looking at the 'then'-clauses, there is an important difference between 'might' and 'would', both expressing modalities.

The auxiliary verb 'would' in premise 2 refers, in connection with the 'if'-clause in the premise, to a state of affairs being either true or false, following from the hypothesis (the 'if'-clause).

However, 'might be' in premise 1 is a so-called alethic modality; alethic modalities indicate in particular the certainty, in terms of logical necessity, possibility, or impossibility of the state of affairs involved. Therefore in the clause 'it might be' we do not refer to a state of affairs being either true or false, but to a possible state of affairs. This implies that in this case 'it' is not just either true or false. Consequently, this modality affects the logical form of the sentence. In fact, alethic modalities are subject of a special type of logic, the so-called 'modal logic'. I will come back to this shortly.

Concerning the use of 'it' occurring six times in the argument, we have already concluded that 'it' stands for a statement, but we then still have three possibilities.

The first possibility is that 'it' refers to a specific statement. When examining the context of the story, the most obvious candidate is: 'Tweedledum and Tweedledee are two characters from a nursery rhyme'.

In the second possibility 'it' is a statement variable, referring to any statement, but the same statement through the whole argument. In terms of the validity of the argument there is no difference between the first and second possibility, since logic is not concerned with the meaning of statements.

The third possibility is that 'it' is a meta-variable that may refer to different statements throughout the argument. This case is considered by Donald Knuth, who presents an exercise ("Make sense of Tweedledee's comment ...") in which he appears to be seeking different "instances" of 'it' in order to come to a sound argument. He presents a "solution by C. Sartena", who "was describing the implication ' $x \rightarrow y$ ', with *it* standing respectively for $y, x, x, y, y, x,$ " but also notes that other solutions are possible [Knuth 2011, p.534]⁹. According to this solution, the argument can then be represented as follows (where ' \rightarrow ' stands for 'implies', ' \neg ' for a negation and ' \therefore ' signifies the conclusion):

$y \rightarrow x$

$x \rightarrow y$

$\therefore \neg y \rightarrow \neg x$

This may be useful as an exercise in a logic book, it is not likely that Carroll intended to let Tweedledee present an exercise instead of presenting an example of sound logic. Also note that the alethic modality of 'might be' is neglected by Knuth.

Let us now consider the logical structure of the argument.

Peter Heath [1974] offers a reconstruction of the argument, replacing 'it' by ' p ' as a statement variable, indicating the same statement through the argument.

This can be formulated as follows:

1. If p is true, it is possible that p is true
2. And if we assume that p is true, then it follows that p is true
3. But in fact p is not true
4. Therefore p is not true

In Heath's words, including his reference to modal logic: "These statements enunciate a cardinal principle of modal inference (if p then ' p is possible') followed by a formalised version of *modus tollens* (if p then p ; $\neg p$; therefore $\neg p$)" [Heath1974, pp.162-163].¹⁰

We can translate this in the following logical form (where ' \diamond ' stands for 'possible'):

1. $p \rightarrow \diamond p$
2. $p \rightarrow p$
3. $\neg p$
4. $\therefore \neg p$

As mentioned by Heath, lines 2, 3, 4 together have the structure of *modus tollens*¹¹, which is:

- $$\begin{array}{l} p \rightarrow q \\ \neg q \\ \therefore \neg p \end{array}$$

When considering Tweedledee's argument we can note two things.

Compared to the *modus tollens* rule, Tweedledee's argument makes ' p ' and ' q ' identical. By replacing ' q ' by ' p ', we get

- $$\begin{array}{l} p \rightarrow p \\ \neg p \\ \therefore \neg p \end{array}$$

This makes the argument a trivial one, but still with the structure of *modus tollens*.

We can also note that, considering the whole argument as *modus tollens*, premise 1 is superfluous¹².

6. *Reductio ad absurdum*?

Summarising our argument thus far:

- Tweedledum and Tweedledee are mirror-image forms of each other, there is no contrast between them, they complement each other.
- Tweedledee's use of the word 'contrariwise' does not indicate any opposition towards Tweedledum, but is a supplement when it is a reaction to Tweedledum's words; its meaning is: 'suppose it would be the opposite way' or 'in the opposite way'.
- Tweedledee's argument is an ostensive definition of logic, an example of logical reasoning.
- 'It is' or 'it is so' means that a statement ('it') 'is the case' or 'is true'.

Following Heath we can formalise Tweedledee's argument as follows:

- $$\begin{array}{l} \text{TW1. } p \rightarrow \diamond p \\ \text{TW2. } p \rightarrow p \\ \text{TW3. } \neg p \\ \text{TW4. } \therefore \neg p \end{array}$$

TW 2-4 have the form of *modus tollens*, albeit a trivial one.

Let us now investigate whether Tweedledee’s argument can be considered to be *reductio ad absurdum*.

The formal, mathematical variant of *reductio ad absurdum* is a mode of argumentation in which the following steps are essential [Suppes 1957, p. 39]:

- I. introducing the denial of the statement that you want to prove as assumption;
 - II. deriving a contradiction from this assumption;
 - III. asserting the desired conclusion as a logical consequence from this contradiction.
- In other words, the original thesis must be accepted because its rejection would lead to a contradiction¹³.

Within these steps, *modus tollens* is often used to derive a contradiction from the assumption (step II). However, in order to speak of a formal *reductio* we also need step I (assuming the denial of what you want to prove) and step III (making the contradiction explicit, as well as the conclusion that you have actually proved what you wanted to prove).

Let me illustrate the *reductio* with an example: we want to prove the statement that the world is round (statement *A*).

Then you suppose that the world is not round (*not-A*). In that case the world is flat and we will fall off the world when walking towards the end (statement *B*).

So we know that *not-A* implies statement *B*, but we also know that *not-B* is the case: these are our premises.

From these premises we will now derive a contradiction which implies that our hypothesis ($\neg A$) is false and, consequently, that *A* is true.

1. $\neg A$	<i>not-A</i>	hypothesis
2. $\neg A \rightarrow B$	<i>not-A</i> implies <i>B</i>	premise
3. $\neg B$	<i>not-B</i>	premise
4. <i>A</i>	<i>not-A</i>	follows from 2,3 by <i>modus tollens</i>
5. $A \wedge \neg A$	<i>A</i> and <i>not-A</i>	1, 4: contradiction
7. <i>A</i>	<i>A</i>	by <i>reductio ad absurdum</i>

Let us compare this with Tweedledee’s argument.

In fact, Tweedledee wants to support Tweedledum’s statement “It isn’t so, nohow”. Let us give this the logical form $\neg \neg C$. With the word ‘contrariwise’ he implicitly introduces the hypothesis $\neg \neg C$ (“Suppose it was the other way around”). So in our example above we replace ‘*A*’ by ‘ $\neg \neg C$ ’.

Furthermore, we also replace ‘*B*’ by ‘ $\neg \neg C$ ’, since ‘it’ refers to the same statement throughout the argument.

1. $\neg \neg C$	contrariwise	hypothesis
2. <i>C</i>		follows from 1
3. $C \rightarrow C$	TW2	premise
4. $\neg C$	TW3	premise
5. $\neg C$	TW4	follows from 3,4 by <i>modus tollens</i>
6. $C \wedge \neg C$		2, 6: contradiction
7. $\neg C$		

So here we have *modus tollens*, preceded by the assumption of the denial ($\neg \neg C$) of the statement ($\neg C$) Tweedledee wants to prove, and followed by the explicit contradiction and inference of the original statement, $\neg C$.

As I argued before [Savenije 2017, p.39], Carroll regularly uses the dialectical variant of *reductio* in his literary work. This dialectical variant is part of informal logic which, contrary to formal logic, takes the content of the statements involved and the context of use into account. It is, therefore, less rigid than the mathematical variant of *reductio*. Many *reductio* arguments in the dialectical variant are quasi-logical arguments: they do resemble formal arguments, but it takes some effort to formalise them. In the dialectical variant, one or more of the formal conditions formulated in the mathematical version are also often left implicit or completely dropped. For example, the contradiction to which the argument leads may be left unstated.

The formalisation of Tweedledee's argument above shows that Tweedledee's sentences that correspond with TW2, TW3 and TW4 can be seen as a quasi-logical application of *modus tollens*. They do resemble *modus tollens*, but it takes some effort to formalise them as such.

When we add the word 'contrariwise', we have step I of a quasi-logical variant of *reductio ad absurdum* as mentioned above, introducing the denial of the statement that Tweedledee wants to prove as assumption. Then TW2-4 can be considered to be step II (deriving a contradiction from the assumption).

Nevertheless, the explicit contradiction and its consequence (step III) are still missing. As mentioned, this is a common feature in the dialectical variant of *reductio*.

Also TW1 ($p \rightarrow \diamond p$) is superfluous as a premise in the argument as analysed by me.

So we may say that what Tweedledee is doing is:

- presenting an argument of which the form can be analysed as the dialectical variant of *reductio ad absurdum*;
- applying the *reductio* structure to statements that make the argument in fact self-evident;
- adding to this an additional, superfluous premise.

7. Squared *reductio*

In addition to the strict, mathematical use and the quasi-logical, dialectical use of *reductio ad absurdum* we have its rhetoric use, which we also find in Carroll's literary work [Savenije 2017, p.41]. In the rhetoric use a point of view is ridiculed or refuted without being mentioned at all: the absurd consequences of a position are shown, but it is not explicitly presented as an attack. It is an indirect method to convey a message to the audience. It cannot be considered to be a quasi-logical argument, because there is no deductive scheme. The argument may be reconstructed, but only by making use of the context, which is not explicitly presented in the text. And of course we run the risk here of giving interpretations that may not be intended by the author.

For example: in *Alice's Adventures in Wonderland* the compulsive moralising by the Duchess is ridiculed in the dialogue ending with "Tut, tut, child!" said the Duchess. 'Everything's got a moral, if only you can find it.'¹⁴

Since Tweedledee's argument is a play with the *reductio* scheme, it may very well be the case that Carroll is using Tweedledee's argument to ridicule something. The question, then, is: what is being ridiculed?

The first option is that he is ridiculing logic. This, however, is not very likely, since Carroll took logic very seriously and, although he focused on logic later on in life, his diaries confirm his early interest in logic. He also wanted to popularise logic and valued the instruction of logic [Moktefi 2010].

A second option would be that he is ridiculing *reductio ad absurdum*. This, too, is not very likely. Carroll made use of *reductio* not only in his literary works, it also played an important role in his works on logic that he would write later. *Reductio* actually forms the basis of his Method of Trees [Abeles 1990]. Furthermore, *reductio* was often used by Euclid who was highly admired by Carroll, who was "the outspoken advocate for Euclid's Elements" [Wilson 2008, p.91].

That leaves us with a third possibility that, by making fun of Tweedledee's argumentation, Carroll is ridiculing Tweedledee as a character, or, more likely, both the Tweedle twins as characters. In fact Tweedledee makes pompous use of a logical rule, making the actual argument look more impressive than what it really is: a trivial argument with a superfluous premise.

In this context the following description by Florence Becker Lennon is relevant: "Alice asks for guidance out of the wood and is answered with irrelevant and trifling animosities, boasting, cowardice - all the vices of the desk soldier. Oxford was biting into his bone and Dodgson was biting back" [Becker Lennon 1945, p.183]. The complexities of the university's organisation certainly irritated Carroll, as we can see from his Oxford Pamphlets [Wakeling 1993].

In this light I would like to conclude that Carroll, by ridiculing the Tweedle twins, is ridiculing the desk soldier, the type he knows from Oxford University, a bureaucratic type, with his love of routines and procedures without seeing the context. This is fully consistent with Tweedledee's pomposity as it comes forward in his pretentious use of a logical rule as an explanation of logic.

So Carroll takes Tweedledee's dialectical use of *reductio ad absurdum* to ridicule the Oxford bureaucrats, through the characters of the Tweedle twins. In other words, he uses the dialectical *reductio* in a way that may be described as a rhetoric use of *reductio*: the rhetoric use of *reductio* applied to its quasi-logical use. We can qualify this as *reductio ad absurdum* of *reductio ad absurdum*, or even: squared **Reductio ad Absurdum**, (*RaA*)².

Notes

¹ *Through the Looking-Glass and what Alice Found There*, Chapter Four: Tweedledum ad Tweedledee.

² Most of the historic details in this paragraph are derived from Lindseth 2009.

³ *Through the Looking-Glass and what Alice Found There*, Chapter Four: Tweedledum and Tweedledee.

⁴ London: Printed for J. Harris, 3rd Edition, 1807.

⁵ According to Everett Bleiler, quoted in Gardner 2015, p.214.

⁶ See, for instance: Baylis & Haggerty 1988, p.1, Centore 1997, p.1, and Englebretsen 1996, p.142.

⁷ Jourdain presents a view on Tweedledee's definition which differs from an ostensive definition, in his work *The Philosophy of Mr. B*rr*nd R*ss*ll* [1918]. This is "a collection of witticisms on philosophy followed by an appendix of abstracts from the writings of Lewis Carroll, in imitation of the structure of Russell's *The Philosophy of Leibniz*." [Grattan-Guinness 1977, p.125]. It was first published in *The Granta* in 1907 and it was the magazine's practice to replace the vowels of the names of persons being satirised by asterisks. According to Jourdain [1918, p.11] "the view that the fundamental principles of logic consist solely of the law of identity was held by Leibniz, Drobisch, Uberweg, and Tweedledee. Tweedledee, it may be remembered, remarked that certain identities *are* logic." This presupposes that Tweedledee's argument consists of identities, which is not the case, as will become clear from the analysis of the argument in the next paragraph. "If it was so, it might be", or in logical form $p \rightarrow \diamond p$, is not an identity: p and $\diamond p$ are not logically equivalent since $\diamond p \rightarrow p$ does not hold in the case p , although possible, is not actually true.

⁸ Jourdain [1918, p. 11] refers to this possibility: "Now, there is some doubt as he [Tweedledee], like Jevons, understood 'are' to mean what mathematicians mean by '=', or, like Schröder and most logicians, to have the same meaning as the relation of subsumption."

⁹ See Knuth 2011, p.79 Exercise 1.15, and p.534 for the solution of the exercise.

¹⁰ Modal logic goes back to Greek logic and Diodorus (late 4th century BC) already claimed that truth entails possibility. Lewis Carroll himself did not engage in modal logic in his logical works.

¹¹ Arguments of the type of *modus ponens* and *modus tollens* already occurred in Greek logic, although these names were attached later [Bobzien 2002].

¹² David Lewis presents an elaborate analysis of counterfactuals, which could also be applied in the formalisation of Tweedledee's substatement "If it were so, it would be". It refers to a condition which is contrary to fact, but only in this case is supposed to be true. Lewis (1973, p.1) introduces the operator ' $\Box \rightarrow$ ':

' $p \Box \rightarrow q$ ' reads as 'If it were the case that p , then it would be the case that q '.

He, then, argues as follows that inference by *modus tollens* on a counterfactual is valid (op.cit., p.36):

1. $p \Box \rightarrow q$ counterfactual
2. $\neg q$ hypothesis
3. $p \rightarrow q$ follows from 1
4. $\neg p \rightarrow \neg q$ contraposition of 3
5. $\neg p$ 4, 2, *modus ponens*.

Following this analysis, lines 2, 3, 4 of Tweedledee's argument may be formalised as

2. $p \Box \rightarrow p$
3. $\neg p$
4. $\therefore \neg p$

Here also we recognise the structure of *modus tollens*, since $p \Box \rightarrow p$ implies $p \rightarrow p$.

¹³ This is based on the law of the excluded middle, which goes back to Aristotle, saying that a statement must be either true or false, but cannot be both true and false. The law of the excluded middle is accepted by most formal logics, however some intuitionist mathematicians do not accept it.

¹⁴ *Alice's Adventures in Wonderland*, Chapter Nine: The Mock Turtle's Story.

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