CONTRARIWISE
***REDUCTIO AD ABSURDUM* IN THE *ALICE* BOOKS**

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Introduction

> 'I mean, what *is* an un-birthday present?'
> 'A present given when it isn't your birthday, of course.'
> Alice considered a little. 'I like birthday presents best,' she said at last.
> 'You don't know what you're talking about!' cried Humpty Dumpty. "How many days are there in a year?'
> 'Three hundred and sixty-five,' said Alice.
> 'And how many birthdays have you?'
> 'One.'
> (….) 'that shows that there are three hundred and sixty-four days when you might get un-birthday presents –'
> 'Certainly,' said Alice.
> 'And only *one* for birthday presents, you know.'[1]

This dialogue, featuring in Lewis Carroll's *Through the Looking-Glass*, provided the inspiration for the Unbirthday Party in the Walt Disney film *Alice in Wonderland.* And it was *The Unbirthday Song* in this film that fascinated me most when I was watching it with my children, about 20 years ago. I must confess that I did not have clear recollections from reading the Alice books in my own youth, but now I was triggered by the challenging combination of logic and absurdity.
This combination can be found throughout the literary works of Lewis Carroll. Already in his youth he argued that a clock that doesn't go at all is to prefer to a clock that loses a minute a day: the first one shows the right time twice a day, while the latter is only right once in two years.[2]

As long as I can remember I adored paradoxes, wordplay and riddles, which all feature abundantly in Carroll's work. I am also gripped by his sense of humour. As I learned more about Lewis Carroll himself, it became apparent why this was the case. Lewis Carroll was a mathematician who had a particular interest in logic. After graduating from high school, I studied mathematics and philosophy and specialised in a combination of logic, mathematics and linguistics.

A well-known feature in logic of absurdity is the argument form called *Reductio ad Absurdum*, in short *reductio*. In this argument form a statement is refuted by demonstrating that it will inevitably lead to absurd consequences.
There are plenty of *reductio* examples in Carroll's work and that is why he has been qualified as a master of the *reductio*.

> 'Speak when you're spoken to!' the Queen sharply interrupted her.
> 'But if everybody obeyed that rule,' said Alice, who was always ready for a little argument, 'and you only spoke when you were spoken to, and the other person always waited for *you* to begin, you see nobody would say anything, so that –' [3]

Lewis Carroll started rather late in his life with his work on logic. While *Alice's Adventures in Wonderland* was published in 1865, his first work on logic dates from 1886. However, his interest in logic is much older than his publications. It was strongly related to his work as mathematics lecturer, and, in his private writings, he sometimes linked logic with religious thought. An important aspect in his contributions to logic was that he was trying to make logic accessible to a wider public. He was an enthusiastic inventor of games and puzzles.

American philosopher W. W. Bartley claims a high place for Lewis Carroll in the history of logic, but this is not shared by everybody.[4] Some say that his logical works are mainly appreciated for their pedagogic and humorous aspect or that they were meant just to further explore his logical intuitions in his literary work. However, there is only limited support for this. The idea that without his mathematical and logical avocation he would be unable to write his Alice books, is now widely accepted.[5]

Common elements in his literary work and his logic can be identified. The *reductio ad absurdum* is one of the common elements and is the focus of this article.
In the next paragraphs I will analyse the concept of *reductio ad absurdum*, look briefly into the use of *Reductio* in Carroll's logic work and describe examples of its occurrence in the *Alice* books. I will thus illustrate not only the importance of logic in Carroll's life, but also the continuity in his way of thinking.

'That's logic'

> 'I know what you're thinking about,' said Tweedledum; 'but it isn't so, nohow.'
> 'Contrariwise,' continued Tweedledee, 'if it was so, it might be; and if it were so, it would be but as it isn'it, it ain't. That's logic.'[6]

Whatever analysis is applied to this dialogue, it is a perfect illustration of what logic is about: the correctness of reasoning. More precisely, logic is the study of methods and principles used in distinguishing correct from incorrect reasoning.
There are several ways in which this study may be performed. For our purpose it is useful to distinguish between formal and informal logic.

Formal logic goes all the way back to Aristotle and aims at providing systematic means for telling whether arguments are valid or invalid, without looking at the content of the elements of the argument. Formal logic is concerned with the form rather than the content of the statements involved and works with its own symbols.

Informal logic came into use in the late 1960s with the purpose to equip students to assess arguments as found in the mass media. Informal logic aims at a set of methods of

evaluating natural language arguments. Therefore, it interprets the use of statements as speech acts in the context of a dialogue.
When arguments derive their persuasive strength from their similarity with well-established formal modes of reasoning, but require some effort of thought to formalise them, we speak of quasi-logical arguments.[7]

Lewis Carroll engaged himself with formal logic, or, as he called it, symbolic logic. Aristotelian logic had remained dominant in England well into the 19th century. During Carroll's lifetime there was a breakthrough to a wider logical structure with the use of algebraic symbols. Carroll's works contributed to this algebra of logic.[8]
Carroll's first book on logic was *The Game of Logic* (1886). It was presented as a game, meant to popularise logic and to be used for instruction.
Ten years later he published *Symbolic Logic*, which was planned to consist of three parts. Part II and III were never published. In 1977 W. W. Bartley III published large surviving fragments of the second part.
Carroll's innovative contributions were the development of visual methods to deal with complex arguments. The first was his diagrammatic method: an improvement of the already existing diagrams of Venn and Boole to handle arguments about classes. The second was his Method of Trees, a mechanical test of the validity of complicated arguments using a formal *reductio ad absurdum* argument.[9]

Carroll had no role in the development of what we call mathematical logic which considers logic as the foundations of mathematics, concerned with consistency proofs and decision procedures. This started with Frege in 1879 and was brought to public notice by Bertrand Russell in the beginning of the 20th century.

Reductio ad absurdum

There appears to be a variety of definitions of *reductio ad absurdum*.
In everyday speech a *reductio ad absurdum* argument is commonly considered to be a debater's technique of arguing that an opponent's position has implications that are bizarre, unacceptable or obviously false.

> 'I *am* real!' said Alice, and began to cry.
> 'You won't make yourself a bit realler by crying,' Tweedledee remarked: 'there's nothing to cry about.'
> 'If I wasn't real,' Alice said – half laughing through her tears, it all seemed so ridiculous – 'I shouldn't be able to cry.'[10]

Looking at the history of philosophy, *reductio ad absurdum* finds its origin in ancient Greek logic, more precisely in *Dialectic*, being at that time a dialogue in which one tries to refute some opponent's assertion.
Zeno of Elea (5th century BC) made extensive use of paradoxes and arguments related to *reductio ad absurdum*. He was the first to realise that it could be systematically employed as a intellectual weapon, although he did not study the logical rules he applied and his arguments did not make explicit the moves of *reductio*. Zeno probably did not create the method himself. It was already known in a rigorous form from early Greek mathematics.

Socrates († 399 BC) also used *reductio*, most of the time not in a rigorous form, looking for possible contradictions in his dialogues.
Aristotle (4th century BC) was the first to devise a system of formal logic, sorting out valid from invalid arguments and using rules that were explicitly formulated with the help of formulae. Among others, Aristotle formulated the principle of non-contradiction, also called the law of the excluded middle, saying that a statement must be either true or false, but cannot be both true and false. Therefore if a statement and its negation can both be derived logically from a premise, it can be concluded that the premise is false. This technique, called proof by contradiction, has formed the basis of *reductio ad absurdum* arguments in formal fields like logic and mathematics. The law of the excluded middle is accepted by most formal logics, however some intuitionist mathematicians do not accept it.

Outside mathematics, *reductio* is not only used to refute a statement or a group of statements, but it is also applied to rules, procedures or politics.[11]

> Alice was rather doubtful whether she ought not to lie down on her face like the three gardeners, but she could not remember ever having heard of such a rule at processions; 'and besides, what would be the use of a procession,' thought she, 'if people had all to lie down on their faces, so that they couldn't see it?' So she stood where she was, and waited.[12]

In this example, Alice is doubting a rule, extrapolates what happens when a rule ('to lie down when a procession passes') is applied and concludes that this leads to an absurdity.

To bring some order in the variety of its applications, a distinction is often made between a strong and weak variant of *reductio ad absurdum.*[13] For our purpose I prefer the distinction between a mathematical and a dialectical variant.[14] As we will see, Carroll used both variants in his works.

The mathematical variant of *reductio*

The mathematical variant is used by mathematicians and formal logicians. Formal logicians concentrate on the form of the argument and define essential steps for the deduction of a conclusion, without looking at the content of the statements involved. For them, *reductio ad absurdum* is a mode of argumentation that seeks to establish a contention by deriving an untenable consequence from its denial, thus arguing that a thesis must be accepted because its rejection would lead to a contradiction.[15] It presumes the law of excluded middle.
In this mode of argumentation the following steps are essential:[16]
1. introduce the denial of the statement that you want to prove as assumption;
2. derive a contradiction from this assumption;
3. assert the desired conclusion as a logical consequence from this contradiction.

I will give a simple example. You want to prove that the earth is round. Since it is either round or flat, you first assume that the world is not round but flat (1). This would imply that we would fall off if we would walk long enough. So it cannot be true that the world is flat. This contradicts our assumption (2). The logical consequence, then, is that the earth is round, since it is either round or flat (3).

A Greek mathematician who made extensive use of *reductio*, was Euclid of Alexandria († 283 BC). In his main work, *Elements,* Euclid presented his geometry in a single, logically coherent framework, deducing his system from a small set of axioms. It served as main textbook for the teaching of geometry until the late 19th century. Carroll was a great admirer of Euclid. In *Euclid and his Modern Rivals* (1879) he defended Euclid in debates with his contemporaries who wanted to replace it by more modern geometry textbooks.

Carroll's most striking use of *reductio ad absurdum* is his Method of Trees in part II of *Symbolic Logic*. The Method of Trees is a mechanical test of the validity of an argument with a possibly large number of premises, using the strict mathematical version of *reductio*. It had a strong visual element, since its form of a logical tree was modelled on a genealogical tree, growing downwards.

In brief, the method works as follows.
First, list the premises and add the denial of the intended conclusion to them.
Secondly, list the possible consequences of the premises in alternative scenarios, the so-called branches of the tree.
Thirdly, try to reduce each of the scenarios to a formal contradiction. If you succeed for each scenario ('each branch is closed'), you have shown that the denial of the conclusion must lead to a contradiction. So the original conclusion must be true.
Carroll's presentation of this method has the form of a soliloquy, simulating a dialogue. This is an illustration of the fact that the value of *reductio ad absurdum* is best appreciated in the context of a dialogue.

The technique, as developed by Carroll in his Method of Trees, has not been used for a long time, but the semantic tableaux of the Dutch logician Evert Beth show a striking resemblance.[17] However, no proof has been found that Beth was inspired by Carroll's work: Carroll's manuscript on the Method of Trees was only discovered in 1977 by Bartley.

The dialectical variant of *reductio*

The dialectical variant is part of informal logic, looking at the content of the statements involved and the context of use. It is less rigid than the mathematical variant and this leads to a different kind of definition of *reductio*, such as: extending the opposition's argument to its logical conclusion an demonstrating the absurdity of that conclusion.[18] The dialectical variant is often used in the context of a dialogue and can be seen as an art, rather than a brute algorithm.[19]

Many *reductio* arguments in the dialectical variant are quasi-logical arguments: they do resemble formal arguments but it takes some effort to formalise them. Carroll uses the dialectical variant of *reductio* in his literary work.
The fact that we are confronted with quasi-logical arguments implies additional complications. Logic plays an important role in both the *Alice* books.[20] We find all sorts of argumentation, valid and invalid. We see arguments that are formally valid, but have

false premises. We also see invalid arguments, more or less disguised as well-established forms of reasoning.

> 'To begin with,' said the Cat, 'a dog's not mad. You grant that?'
> 'I suppose so,' said Alice.
> 'Well, then,' the Cat went on, 'you see a dog growls when it's angry, and wags its tail when it's pleased. Now I growl when I'm pleased, and wag my tail when I'm angry. Therefore, I'm mad.'[21]

Another complication is that we also find argumentations that cannot be called valid, because one or more statements in it are meaningless: it is impossible to find out whether what they express is true or not. This is caused especially by the fact that Carroll experimented in the *Alice* books with nonsense prose and verse, producing meaningless statements. Important elements of the repertoire of nonsense are: puns such as homophones and wordplay, portmanteau (a word in which two meanings are melted into one, such as brunch), neologism (a new word, for instance to denote a non-existent creature, such as the Jabberwock).

> 'Take some more tea,' the March Hare said to Alice, very earnestly.
> 'I've had nothing yet,' Alice replied in an offended tone: 'so I can't take more.'
> 'You mean you can't take *less*,' said the Hatter: 'it's very easy to take *more* than nothing.'[22]

Let us now turn to some *reductio* examples in the *Alice* books.
In the dialectical variant, often one or more of the formal conditions formulated in the mathematical version are left implicit or completely dropped. The following examples refer to different ways in which the dialectical variant may diverge from the more rigid mathematical variant.

<u>The contradiction to which the argument leads is left unstated.</u>
The argument ends merely in a conclusion which the author takes to be false and does not explicate the contradiction to which it leads.[23] In our example of the earth being round, the argument would be less explicit and run as follows: 'The earth is either round or flat, but it cannot be flat, since in that case we would fall off.'

> (….) splash! she was up to her chin in salt-water. Her first idea was that she had fallen somehow into sea, 'and in that case I can go back by railway,' she said to herself. (Alice had been at the seaside once in her life, and had come to the general conclusion that, wherever you go to on the English coast, you find a number of bathing-machines in the sea, some children digging in the sand with wooden spades, then a row of lodging-houses, and behind them a railway station.) However, she soon made out that she was in the pool of tears which she had wept when she was nine feet high.[24]

Alice is involved in a soliloquy as she wonders in what kind of water she is fallen. She looks at the possible consequences from being in the sea and concludes from this that she is not in the sea.

<u>The conclusion is not necessarily false but implausible.</u>

It is shown that the opponent's position is untenable, but not necessarily with a formal contradiction or even with a falsity. Its consequence may be just implausible or not in accordance with general convention.

> At this moment the King (....) called out 'Silence!', and read out from his book, 'Rule Forty-two. *All persons more than a mile high to leave the court.*'
> Everybody looked at Alice.
> '*I'm* not a mile high,' said Alice.
> 'You are,' said the King.
> 'Nearly two miles high,' added the Queen.
> 'Well, I sha'n't go, at any rate,' said Alice: 'besides, that's not a regular rule: you invented it just now.'
> 'It's the oldest rule in the book,' said the King.
> 'Then it ought to be Number One,' said Alice.[25]

<u>No use is made of the law of excluded middle.</u>
An argument may also show that a certain thesis cannot hold, without implying that its negation is true. It simply proves the untenability of the opponent's view and thus makes no use of the law of excluded middle.[26]

> 'If everybody minded their own business,' the Duchess said, in a hoarse growl, 'the world would go round a deal faster than it does.'
> "Which would not be an advantage,' said Alice, who felt glad to get an opportunity of showing off a little of her knowledge. "Just think what work it would make with the day and night! You see the earth takes twenty-four hours to turn round on its axis –'
> 'Talking of axes,' said the Duchess, 'chop off her head!'[27]

Rhetorical use of *reductio*

In both the mathematical and dialectical variant *reductio* is an argument form. Logical as well as quasi-logical arguments have a deductive scheme which corresponds with or strongly resembles well-established formal modes of reasoning. In quasi-logical arguments certain formal elements are left implicit or completely dropped. However, there is always an opposition's viewpoint that is attacked and refuted.
In the *Alice* books we also find instances where a point of view is ridiculed or refuted without being mentioned at all. The absurd consequences of a position are shown, but it is not explicitly presented as an attack on this position. One needs to understand the context of the story (for instance the audience, or the social circumstances) to identify these occurrences.
I will call this the *rhetoric* variant of *reductio*. Rhetoric is the means by which the writer makes his vision known to the reader and persuades him/her of its validity.[28] We can, then, consider this use of *reductio* as a narrative technique, used as an indirect method to convey a message to the audience. It cannot be considered to be a quasi-logical argument, because there is no deductive scheme. The argument may be reconstructed but only by making use of the context, which is not presented or referred to in the text. And of course we run the risk here of giving interpretations that may not be intended by the author. However, on the other hand, it was Carroll who said: 'Words mean more than

we mean to express when we use them; so a whole book ought to mean a great deal more than the writer meant.'[29]

The following example is from *Alice's Adventures in Wonderland* and is about the Duchess's moralising.

> [Alice] had quite forgotten the Duchess by this time, and was a little startled when she heard her voice close to her ear. 'You're thinking about something, my dear, and that makes you forget to talk. I can't tell just now what the moral of that is, but I shall remember it in a bit.'
> 'Perhaps it hasn't one,' Alice ventured to remark.
> 'Tut, tut, tut, child!' said the Duchess. 'Everything's got a moral, if only you can find it.'[30]

The compulsive moralising of the Duchess throughout the story can be seen as *reductio ad absurdum* of the conventional moralising found in popular children's books in the 19th century: Carroll intends to show how this systematic moralising leads to absurd consequences in the eyes of the audience, the children.

Finally, there is at least one interpretation of the *Alice* books that argues that *reductio* can be considered to be a narrative framework for *Alice's Adventures in Wonderland* as a whole, presenting a basic structure that underlies the whole story. This has to do with Carroll's views as mathematician.
Mathematics in Victorian England was based on Euclid's *Elements*, and defined the way in which space and reality were conceived. Religion and mathematical truth were intertwined. But elsewhere in Europe a new paradigm of mathematics was developed. Truth was no longer seen as something transcendental and absolute, but as something that could stand against all empirical claims to the contrary. These new mathematics involved, for instance, negative and imaginary numbers and also n-dimensional spaces. Melanie Bayley,[31] Jennifer Duggan[32] and Helena Pycior[33] present a number of examples to illustrate that the story of *Alice's Adventures in Wonderland* might be seen as *reductio ad absurdum* by which Carroll wanted to show that the new mathematics were impossible in reality. When the new mathematics would be true, there would be chaos, a shifting reality in which logic is really illogic and nothing makes much sense. In short, new mathematics was symbolised as a 'curious dream' and nothing more.
The debate about this view on *Alice's Adventures in Wonderland* goes beyond the scope of this article. I will, therefore, confine myself to referring to a recent article by Francine Abeles who argues that Carroll certainly was open to the new mathematical ideas of his time, implying that the interpretation of Bayley et al. cannot be maintained.[34]

Conclusion

In the previous chapters I have illustrated the remarkable role of *reductio ad absurdum* in the work of Lewis Carroll. His predilection for this argument form can be traced back to his position as mathematician, being an adept of Euclid, from 1855 on. It became even more apparent in his *Symbolic Logic Part II* on which he was working when he died in 1898. Here he applied the mathematical variant of *reductio ad absurdum* in his Method of Trees. In between these years he wrote the *Alice* books (published in 1865 and 1871), where he applied the dialectical variant of *reductio*.

Clearly, logic was important for Lewis Carroll throughout his life. His use of *reductio ad absurdum,* as illustrated in this article, contradicts the opinion that his logic was only meant to explore further his logical intuitions in his literary work and supports the view that he was a logician in his own right.

Footnotes

[1] *Through The Looking Glass and What Alice Found There*. Chapter Six: Humpty Dumpty.
[2] *The Rectory Umbrella and Mischmasch*. Difficulties No 2.
[3] *Through The Looking-Glass and What Alice Found There.* Chapter Nine: Queen Alice.
[4] Bartley, III, William Warren (ed.) (1977). *Lewis Carroll's Symbolic Logic.* New York: Clarkson N. Potter.
[5] Moktefi, Amirouche (2005). 'How Did Lewis Carroll Become a Logician?' Conference paper.
https://www.academia.edu/26403294/How_did_Lewis_Carroll_become_a_logician.
[6] *Through The Looking Glass and What Alice Found There*. Chapter Four: Tweedledum and Tweedledee.
[7] Perelman, Chaïm & Olbrechts-Tyteca, Lucie (1969). *The new rhetoric: A treatise on argumentation*. Notre Dame: University of Notre Dame Press.
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[10] *Through The Looking Glass and What Alice Found There*. Chapter Four: Tweedledum and Tweedledee.
[11] Rescher, Nicholas (2002). 'Reductio ad Absurdum'. *The Internet Encyclopedia of Philosophy*, ISSN 2161-0002, http://www.iep.utm.edu/.
[12] *Alice's Adventures in Wonderland*. Chapter Eight: The Queen's Croquet-Ground.
[13] See for instance: Ryle, Gilbert (1945). *Philosophical Arguments. An Inaugural Lecture.* Oxford: At the Clarendon Press.
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[20] Patten, Bernard (2009). *The Logic of Alice. Clear Thinking in Wonderland.* New York: Prometheus Books.

[21] *Alice's Adventures in Wonderland.* Chapter six: Pig and Pepper.
[22] *Alice's Adventures in Wonderland.* Chapter seven: A Mad Tea Party.
[23] Nolt, John Eric (1984). *Informal Logic. Possible Worlds and Imagination*. New York: McGraw-Hill.
[24] *Alice's Adventures in Wonderland.* Chapter Two: The Pool of Tears.
[25] *Alice's Adventures in Wonderland.* Chapter Twelve: Alice's Evidence.
[26] Schwed, Menasche (1999). 'What makes the Reductio ad Absurdum an Important Tool for Rationality?' In F.H. van Eemeren, R. Grootendorst, J.A. Blair & C.A. Willard (eds.), *Proceedings of the Fourth Conference of the International Society for the Study of Argumentation.* Amsterdam: Sicsat, pp. 734-735.
[27] *Alice's Adventures in Wonderland.* Chapter six: Pig and Pepper.
[28] Booth, Wayne C. (1961). *The Rhetoric of Fiction*. Chicago: The University of Chicago Press.
[29] Cited in: Gardner, Martin (1973). *The Annotated Snark*. Penguin Books, p. 22.
[30] *Alice's Adventures in Wonderland*. Chapter Nine: The Mock Turtle's Story.
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